Problem 27

Consider the equation $ay'' + by' + cy = d$, where a, b, c, and d are constants.

(a) Find all equilibrium, or constant, solutions of this differential equation.

(b) Let y_e denote an equilibrium solution, and let $Y = y - y_e$. Thus Y is the deviation of a solution y from an equilibrium solution. Find the differential equation satisfied by Y.

Solution

Part (a)

To find the equilibrium solutions, plug in $y = y_e$ into the ODE.

$$\underbrace{a(y_e)''}_{=0} + \underbrace{b(y_e)'}_{=0} + cy_e = d$$

Therefore,

$$y_e = \frac{d}{c}.$$

Part (b)

Make a change of variables $Y = y - y_e$. Differentiate both sides of it with respect to t to find what y' and y'' are in terms of this new variable.

$$Y' = y'$$
$$Y'' = y''$$

Consequently, the ODE that Y satisfies is

$$aY'' + bY' + c(Y + y_e) = d$$

$$aY'' + bY' + cY + cy_e = d$$

$$aY'' + bY' + cY = d - cy_e$$

$$aY'' + bY' + cY = d - c\left(\frac{d}{c}\right)$$

$$aY'' + bY' + cY = 0.$$