Problem 4

In each of Problems 1 through 8, find the general solution of the given differential equation.

$$2y'' - 3y' + y = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2 e^{rt}$$

Substitute these expressions into the ODE.

$$2(r^2 e^{rt}) - 3(re^{rt}) + e^{rt} = 0$$

Divide both sides by e^{rt}.

$$2r^2 - 3r + 1 = 0$$

$$(2r - 1)(r - 1) = 0$$

$$r = \left\{\frac{1}{2}, 1\right\}$$

Two solutions to the ODE are $y = e^{t/2}$ and $y = e^t$. Therefore, the general solution is

$$y(t) = C_1 e^{t/2} + C_2 e^t,$$

a linear combination of the two.