

Problem 5

In each of Problems 1 through 8, find the general solution of the given differential equation.

$$y'' + 5y' = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2e^{rt}$$

Substitute these expressions into the ODE.

$$r^2e^{rt} + 5(re^{rt}) = 0$$

Divide both sides by e^{rt} .

$$r^2 + 5r = 0$$

$$r(r + 5) = 0$$

$$r = \{-5, 0\}$$

Two solutions to the ODE are $y = e^{-5t}$ and $y = e^0 = 1$. Therefore, the general solution is

$$y(t) = C_1e^{-5t} + C_2,$$

a linear combination of the two.