Problem 7

In each of Problems 1 through 8, find the general solution of the given differential equation.

$$y'' - 9y' + 9y = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2 e^{rt}$$

Substitute these expressions into the ODE.

$$r^2 e^{rt} - 9(re^{rt}) + 9(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$r^2 - 9r + 9 = 0$$

$$r = \frac{9 \pm \sqrt{81 - 4(9)(1)}}{2} = \frac{9 \pm \sqrt{45}}{2} = \frac{9 \pm 3\sqrt{5}}{2}$$

$$r = \left\{ \frac{9 - 3\sqrt{5}}{2}, \frac{9 + 3\sqrt{5}}{2} \right\}$$

Two solutions to the ODE are

$$y = \exp\left(\frac{9 - 3\sqrt{5}}{2} t\right) \quad \text{and} \quad y = \exp\left(\frac{9 + 3\sqrt{5}}{2} t\right).$$

Therefore, the general solution is

$$y(t) = C_1 \exp\left(\frac{9 - 3\sqrt{5}}{2} t\right) + C_2 \exp\left(\frac{9 + 3\sqrt{5}}{2} t\right),$$

a linear combination of the two.