Problem 8

In each of Problems 1 through 8, find the general solution of the given differential equation.

$$y'' - 2y' - 2y = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \to \quad y' = re^{rt} \quad \to \quad y'' = r^2e^{rt}$$

Substitute these expressions into the ODE.

$$r^2e^{rt} - 2(re^{rt}) - 2(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$r^2 - 2r - 2 = 0$$

$$r = \frac{2 \pm \sqrt{4 + 4(2)(1)}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

$$r = \left\{1 - \sqrt{3}, 1 + \sqrt{3}\right\}$$

Two solutions to the ODE are $y = e^{(1-\sqrt{3})t}$ and $y = e^{(1+\sqrt{3})t}$. Therefore, the general solution is

$$y(t) = C_1e^{(1-\sqrt{3})t} + C_2e^{(1+\sqrt{3})t},$$

a linear combination of the two.