Problem 7

In each of Problems 7 through 12, determine the longest interval in which the given initial value problem is certain to have a unique twice-differentiable solution. Do not attempt to find the solution.

$$ty'' + 3y = t, \qquad y(1) = 1, \quad y'(1) = 2$$

Solution

Divide both sides of the ODE by t so that the coefficient of y'' is 1.

$$y'' + \frac{3}{t}y = 1$$

There is a point of discontinuity at $t = 0$, which means the interval in which the general solution is unique and twice-differentiable is either $-\infty < t < 0$ or $0 < t < \infty$. Because y and y' are prescribed at $t = 1$, the general solution is unique and twice-differentiable on $0 < t < \infty$.