Problem 11

In each of Problems 7 through 12, determine the longest interval in which the given initial value problem is certain to have a unique twice-differentiable solution. Do not attempt to find the solution.

$$(x-3)y'' + xy' + (\ln|x|)y = 0, \qquad y(1) = 0, \quad y'(1) = 1$$

Solution

Divide both sides of the ODE by $x - 3$ so that the coefficient of y'' is 1.

$$y'' + \frac{x}{x-3}y' + \frac{\ln|x|}{x-3}y = 0$$

There are points of discontinuity at $x = 0$ and $x = 3$, which means the interval in which the general solution is unique and twice-differentiable is either $-\infty < x < 0$ or $0 < x < 3$ or $3 < x < \infty$. Because y and y' are prescribed at $x = 1$, the general solution is unique and twice-differentiable on $0 < x < 3$.