

Problem 24

In each of Problems 24 through 27, verify that the functions y_1 and y_2 are solutions of the given differential equation. Do they constitute a fundamental set of solutions?

$$y'' + 4y = 0; \quad y_1(t) = \cos 2t, \quad y_2(t) = \sin 2t$$

Solution

Check that y_1 is a solution of the ODE.

$$y_1'' + 4y_1 \stackrel{?}{=} 0$$

$$\frac{d^2}{dt^2}(\cos 2t) + 4(\cos 2t) \stackrel{?}{=} 0$$

$$-4 \cos 2t + 4 \cos 2t \stackrel{?}{=} 0$$

$$0 = 0$$

Now check that y_2 is a solution of the ODE.

$$y_2'' + 4y_2 \stackrel{?}{=} 0$$

$$\frac{d^2}{dt^2}(\sin 2t) + 4(\sin 2t) \stackrel{?}{=} 0$$

$$-4 \sin 2t + 4 \sin 2t \stackrel{?}{=} 0$$

$$0 = 0$$

Calculate $W(y_1, y_2)$, the Wronskian of y_1 and y_2 .

$$\begin{aligned} W(y_1, y_2) &= \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \\ &= \begin{vmatrix} \cos 2t & \sin 2t \\ -2 \sin 2t & 2 \cos 2t \end{vmatrix} \\ &= \cos 2t(2 \cos 2t) - \sin 2t(-2 \sin 2t) \\ &= 2 \cos^2 2t + 2 \sin^2 2t \\ &= 2(\cos^2 2t + \sin^2 2t) \\ &= 2 \end{aligned}$$

Since $W(y_1, y_2) \neq 0$, y_1 and y_2 form a fundamental set of solutions.