

## Problem 28

Consider the equation  $y'' - y' - 2y = 0$ .

- (a) Show that  $y_1(t) = e^{-t}$  and  $y_2(t) = e^{2t}$  form a fundamental set of solutions.
- (b) Let  $y_3(t) = -2e^{2t}$ ,  $y_4(t) = y_1(t) + 2y_2(t)$ , and  $y_5(t) = 2y_1(t) - 2y_3(t)$ . Are  $y_3(t)$ ,  $y_4(t)$ , and  $y_5(t)$  also solutions of the given differential equation?
- (c) Determine whether each of the following pairs forms a fundamental set of solutions:  
 $[y_1(t), y_3(t)]$ ;  $[y_2(t), y_3(t)]$ ;  $[y_1(t), y_4(t)]$ ;  $[y_4(t), y_5(t)]$ .