

**Problem 37**

If  $f$ ,  $g$ , and  $h$  are differentiable functions, show that  $W(fg, fh) = f^2W(g, h)$ .

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**Solution**

Calculate the Wronskian of  $fg$  and  $fh$ .

$$\begin{aligned}W(fg, fh) &= \begin{vmatrix} fg & fh \\ (fg)' & (fh)' \end{vmatrix} \\&= \begin{vmatrix} fg & fh \\ f'g + fg' & f'h + fh' \end{vmatrix} \\&= fg(f'h + fh') - fh(f'g + fg') \\&= \cancel{ff'gh} + f^2gh' - \cancel{ff'gh} - f^2g'h \\&= f^2(gh' - g'h) \\&= f^2 \begin{vmatrix} g & h \\ g' & h' \end{vmatrix} \\&= f^2W(g, h)\end{aligned}$$