

## Problem 41

**Exact Equations.** The equation

$$P(x)y'' + Q(x)y' + R(x)y = 0$$

is said to be exact if it can be written in the form

$$[P(x)y']' + [f(x)y]' = 0,$$

where  $f(x)$  is to be determined in terms of  $P(x)$ ,  $Q(x)$ , and  $R(x)$ . The latter equation can be integrated once immediately, resulting in a first order linear equation for  $y$  that can be solved as in Section 2.1. By equating the coefficients of the preceding equations and then eliminating  $f(x)$ , show that a necessary condition for exactness is

$$P''(x) - Q'(x) + R(x) = 0.$$

It can be shown that this is also a sufficient condition.