

Problem 49

In each of Problems 47 through 49, use the result of Problem 46 to find the adjoint of the given differential equation.

$$y'' - xy = 0, \quad \text{Airy's equation}$$

Solution

To make the ODE exact, multiply both sides by an integrating factor $\mu = \mu(x)$.

$$\mu(x)y'' - x\mu(x)y = 0 \tag{1}$$

Now that it's exact, it can be written in the form,

$$[\mu(x)y']' + [f(x)y]' = 0.$$

Expand the left side.

$$\mu'(x)y' + \mu(x)y'' + f'(x)y + f(x)y' = 0$$

Factor it now.

$$\mu(x)y'' + [\mu'(x) + f(x)]y' + f'(x)y = 0$$

Equate the coefficients with those of equation (1).

$$\begin{aligned} \mu'(x) + f(x) &= 0 \\ f'(x) &= -x\mu(x) \end{aligned}$$

Differentiate both sides of the first equation with respect to x .

$$\mu''(x) + f'(x) = 0$$

Substitute $-x\mu(x)$ for $f'(x)$.

$$\mu''(x) - x\mu(x) = 0$$