

## Problem 5

In each of Problems 1 through 6, find the Wronskian of the given pair of functions.

$$e^t \sin t, \quad e^t \cos t$$

---

### Solution

The Wronskian of these two functions is

$$\begin{aligned} W &= \begin{vmatrix} e^t \sin t & e^t \cos t \\ \frac{d}{dt}(e^t \sin t) & \frac{d}{dt}(e^t \cos t) \end{vmatrix} \\ &= \begin{vmatrix} e^t \sin t & e^t \cos t \\ e^t \sin t + e^t \cos t & e^t \cos t - e^t \sin t \end{vmatrix} \\ &= e^t \sin t(e^t \cos t - e^t \sin t) - e^t \cos t(e^t \sin t + e^t \cos t) \\ &= e^{2t} \sin t \cos t - e^{2t} \sin^2 t - e^{2t} \sin t \cos t - e^{2t} \cos^2 t \\ &= -e^{2t} \sin^2 t - e^{2t} \cos^2 t \\ &= -e^{2t}(\sin^2 t + \cos^2 t) \\ &= -e^{2t}. \end{aligned}$$