Problem 14

In each of Problems 7 through 16, find the general solution of the given differential equation.

$$9y'' + 9y' - 4y = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \to \quad y' = re^{rt} \quad \to \quad y'' = r^2 e^{rt}$$

Substitute these expressions into the ODE.

$$9(r^2 e^{rt}) + 9(re^{rt}) - 4(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$9r^2 + 9r - 4 = 0$$

$$(3r + 4)(3r - 1) = 0$$

$$r = \left\{-\frac{4}{3}, \frac{1}{3}\right\}$$

Two solutions to the ODE are $y = e^{-4t/3}$ and $y = e^{t/3}$, so the general solution is

$$y(t) = C_1 e^{-4t/3} + C_2 e^{t/3},$$

a linear combination of the two.