Problem 29

Using Euler's formula, show that

$$\cos t = (e^{it} + e^{-it})/2, \qquad \sin t = (e^{it} - e^{-it})/2i.$$

Solution

Euler's formula is

$$e^{it} = \cos t + i \sin t. \tag{1}$$

Use $-t$ for t to obtain a second formula.

$$\begin{aligned} e^{i(-t)} &= \cos(-t) + i \sin(-t) \\ e^{-it} &= \cos t - i \sin t \end{aligned} \tag{2}$$

Add the respective sides of equations (1) and (2).

$$e^{it} + e^{-it} = 2 \cos t$$

Therefore,

$$\cos t = \frac{e^{it} + e^{-it}}{2}.$$

Subtract the respective sides of equations (1) and (2).

$$e^{it} - e^{-it} = 2i \sin t$$

Therefore,

$$\sin t = \frac{e^{it} - e^{-it}}{2i}.$$