Problem 34

Euler Equations. An equation of the form

$$t^2\frac{d^2y}{dt^2} + \alpha t\frac{dy}{dt} + \beta y = 0, \qquad t > 0, \tag{ii}$$

where α and β are real constants, is called an Euler equation.

(a) Let $x = \ln t$ and calculate dy/dt and d^2y/dt^2 in terms of dy/dx and d^2y/dx^2.

(b) Use the results of part (a) to transform Eq. (ii) into

$$\frac{d^2y}{dx^2} + (\alpha - 1)\frac{dy}{dx} + \beta y = 0. \tag{iii}$$

Observe that Eq. (iii) has constant coefficients. If $y_1(x)$ and $y_2(x)$ form a fundamental set of solutions of Eq. (iii), then $y_1(\ln t)$ and $y_2(\ln t)$ form a fundamental set of solutions of Eq. (ii).