

Problem 5

In each of Problems 1 through 6, use Euler's formula to write the given expression in the form $a + ib$.

$$2^{1-i}$$

Solution

Euler's formula states that $e^{ix} = \cos x + i \sin x$.

$$\begin{aligned} 2^{1-i} &= e^{\ln 2^{1-i}} \\ &= e^{(1-i) \ln 2} \\ &= e^{\ln 2 - i \ln 2} \\ &= e^{\ln 2} e^{-i \ln 2} \\ &= 2[\cos(-\ln 2) + i \sin(-\ln 2)] \\ &= 2[\cos(\ln 2) - i \sin(\ln 2)] \\ &= 2 \cos(\ln 2) - 2i \sin(\ln 2) \\ &\approx 1.54 - 1.28i \end{aligned}$$