

Problem 7

In each of Problems 1 through 10, find the general solution of the given differential equation.

$$4y'' + 17y' + 4y = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2e^{rt}$$

Substitute these expressions into the ODE.

$$4(r^2e^{rt}) + 17(re^{rt}) + 4(e^{rt}) = 0$$

Divide both sides by e^{rt} .

$$4r^2 + 17r + 4 = 0$$

$$(4r + 1)(r + 4) = 0$$

$$r = \left\{ -4, -\frac{1}{4} \right\}$$

Two solutions to the ODE are $y = e^{-4t}$ and $y = e^{-t/4}$, so the general solution is a linear combination of the two.

$$y(t) = C_1e^{-4t} + C_2e^{-t/4}$$