

Problem 33

In each of Problems 33 through 36, use the method of Problem 32 to find a second independent solution of the given equation.

$$t^2 y'' + 3ty' + y = 0, \quad t > 0; \quad y_1(t) = t^{-1}$$

Solution

Divide both sides of the ODE by t^2 to put it in the proper form.

$$y'' + \frac{3}{t}y' + \frac{1}{t^2}y = 0$$

Use the result of Problem 32 to determine a second solution.

$$\begin{aligned} y_2(t) &= y_1(t) \int^t \frac{1}{[y_1(r)]^2} \exp \left[- \int^r p(s) ds \right] dr \\ &= t^{-1} \int^t \frac{1}{(r^{-1})^2} \exp \left(- \int^r \frac{3}{s} ds \right) dr \\ &= t^{-1} \int^t r^2 \exp(-3 \ln r) dr \\ &= t^{-1} \int^t r^2 e^{\ln r^{-3}} dr \\ &= t^{-1} \int^t r^2 (r^{-3}) dr \\ &= t^{-1} \int^t \frac{dr}{r} \\ &= t^{-1} \ln t \end{aligned}$$