

## Problem 20

Problems 20 through 22 indicate other ways of finding the second solution when the characteristic equation has repeated roots.

- (a) Consider the equation  $y'' + 2ay' + a^2y = 0$ . Show that the roots of the characteristic equation are  $r_1 = r_2 = -a$ , so that one solution of the equation is  $e^{-at}$ .
- (b) Use Abel's formula [Eq. (23) of Section 3.2] to show that the Wronskian of any two solutions of the given equation is

$$W(t) = y_1(t)y_2'(t) - y_1'(t)y_2(t) = c_1e^{-2at},$$

where  $c_1$  is a constant.

- (c) Let  $y_1(t) = e^{-at}$  and use the result of part (b) to obtain a differential equation satisfied by a second solution  $y_2(t)$ . By solving this equation, show that  $y_2(t) = te^{-at}$ .