

## Problem 21

Suppose that  $r_1$  and  $r_2$  are roots of  $ar^2 + br + c = 0$  and that  $r_1 \neq r_2$ ; then  $\exp(r_1 t)$  and  $\exp(r_2 t)$  are solutions of the differential equation  $ay'' + by' + cy = 0$ . Show that  $\phi(t; r_1, r_2) = [\exp(r_2 t) - \exp(r_1 t)] / (r_2 - r_1)$  is also a solution of the equation for  $r_2 \neq r_1$ . Then think of  $r_1$  as fixed, and use L'Hôpital's rule to evaluate the limit of  $\phi(t; r_1, r_2)$  as  $r_2 \rightarrow r_1$ , thereby obtaining the second solution in the case of equal roots.