Problem 35

In this problem we indicate an alternative procedure[7] for solving the differential equation

$$y'' + by' + cy = (D^2 + bD + c)y = g(t), \tag{i}$$

where b and c are constants, and D denotes differentiation with respect to t. Let r_1 and r_2 be the zeros of the characteristic polynomial of the corresponding homogeneous equation. These roots may be real and different, real and equal, or conjugate complex numbers.

(a) Verify that Eq. (i) can be written in the factored form

$$(D - r_1)(D - r_2)y = g(t),$$

where $r_1 + r_2 = -b$ and $r_1 r_2 = c$.

(b) Let $u = (D - r_2)y$. Then show that the solution of Eq (i) can be found by solving the following two first order equations:

$$(D - r_1)u = g(t), \qquad (D - r_2)y = u(t).$$

TYPO: In part (b), "Eq (i)" should read "Eq. (i)" to be consistent.

[7] R. S. Luthar, "Another Approach to a Standard Differential Equation," *Two Year College Mathematics Journal 10* (1979), pp. 200–201. Also see D. C. Sandell and F. M. Stein, "Factorization of Operators of Second Order Linear Homogeneous Ordinary Differential Equations," *Two Year College Mathematics Journal 8* (1977), pp. 132–141, for a more general discussion of factoring operators.