

Problem 21

Show that the solution of the initial value problem

$$L[y] = y'' + p(t)y' + q(t)y = g(t), \quad y(t_0) = y_0, \quad y'(t_0) = y'_0 \quad (\text{i})$$

can be written as $y = u(t) + v(t)$, where u and v are solutions of the two initial value problems

$$L[u] = 0, \quad u(t_0) = y_0, \quad u'(t_0) = y'_0, \quad (\text{ii})$$

$$L[v] = g(t), \quad v(t_0) = 0, \quad v'(t_0) = 0, \quad (\text{iii})$$

respectively. In other words, the nonhomogeneities in the differential equation and in the initial conditions can be dealt with separately. Observe that u is easy to find if a fundamental set of solutions of $L[u] = 0$ is known.