

Problem 28

The method of reduction of order (Section 3.4) can also be used for the nonhomogeneous equation

$$y'' + p(t)y' + q(t)y = g(t), \quad (\text{i})$$

provided one solution y_1 of the corresponding homogeneous equation is known. Let $y = v(t)y_1(t)$ and show that y satisfies Eq. (i) if v is a solution of

$$y_1(t)v'' + [2y_1'(t) + p(t)y_1(t)]v' = g(t). \quad (\text{ii})$$

Equation (ii) is a first order linear equation for v' . By solving this equation, integrating the result, and then multiplying by $y_1(t)$, you can find the general solution of Eq. (i).