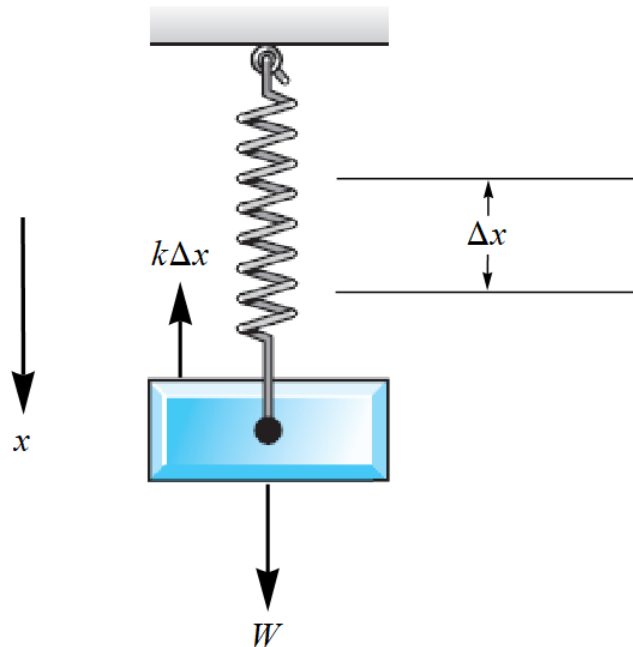


Problem 17

A mass weighing 8 lb stretches a spring 1.5 in. The mass is also attached to a damper with coefficient γ . Determine the value of γ for which the system is critically damped; be sure to give the units for γ .

Solution

Draw a free-body diagram of the mass in equilibrium.



Use the fact that the force of gravity balances the force of the spring to determine k , the spring constant.

$$\begin{aligned} W &= k\Delta x \\ 8 \text{ lb} &= k \left(1.5 \cancel{\text{in}} \times \frac{1 \text{ ft}}{12 \cancel{\text{in}}} \right) \\ k &= 64 \frac{\text{lb}}{\text{ft}} \end{aligned}$$

Critical damping occurs when the ratio of γ^2 to $4km$ is 1.

$$\begin{aligned} \frac{\gamma^2}{4km} = 1 &\quad \rightarrow \quad \gamma^2 = 4km \quad \rightarrow \quad \gamma = \sqrt{4km} \\ &= \sqrt{4k \frac{W}{g}} \\ &= \sqrt{4 \left(64 \frac{\text{lb}}{\text{ft}} \right) \frac{8 \text{ lb}}{32.2 \frac{\text{ft}}{\text{s}^2}}} \\ &\approx 7.98 \frac{\text{lb} \cdot \text{s}}{\text{ft}} \end{aligned}$$