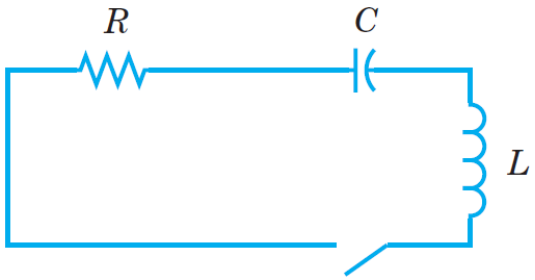
Problem 18

If a series circuit has a capacitor of $C = 0.8 \times 10^{-6}$ F and an inductor of $L = 0.2$ H, find the resistance R so that the circuit is critically damped.

Solution

Draw a schematic of the RLC circuit in series.



Assume the circuit is closed at $t = 0$. Apply Faraday's law to obtain the governing equation for the current.

$$\sum V = -L\frac{di}{dt}$$

The only potential drops occur over the resistor and the capacitor.

$$iR + \frac{q}{C} = -L\frac{di}{dt}$$

Write $i = dq/dt = q'$.

$$Rq' + \frac{q}{C} = -Lq''$$

$$Lq'' + Rq' + \frac{q}{C} = 0$$

Critical damping occurs when the ratio of R^2 to $4L/C$ is 1.

$$\frac{R^2}{\frac{4L}{C}} = 1 \quad \rightarrow \quad R^2 = \frac{4L}{C} \quad \rightarrow \quad R = \sqrt{\frac{4L}{C}}$$

$$= \sqrt{\frac{4(0.2 \text{ H})}{0.8 \times 10^{-6} \text{ F}}}$$

$$= 1000 \ \Omega$$