Problem 7

(a) Find the solution of Problem 5.

(b) Plot the graph of the solution.

(c) If the given external force is replaced by a force $4\sin\omega t$ of frequency ω, find the value of ω for which resonance occurs.

Solution

The initial value problem in Problem 5 was

$$mx'' + kx = W + 2\cos 3t, \qquad x(0) = \frac{7}{24}, \quad x'(0) = 0,$$

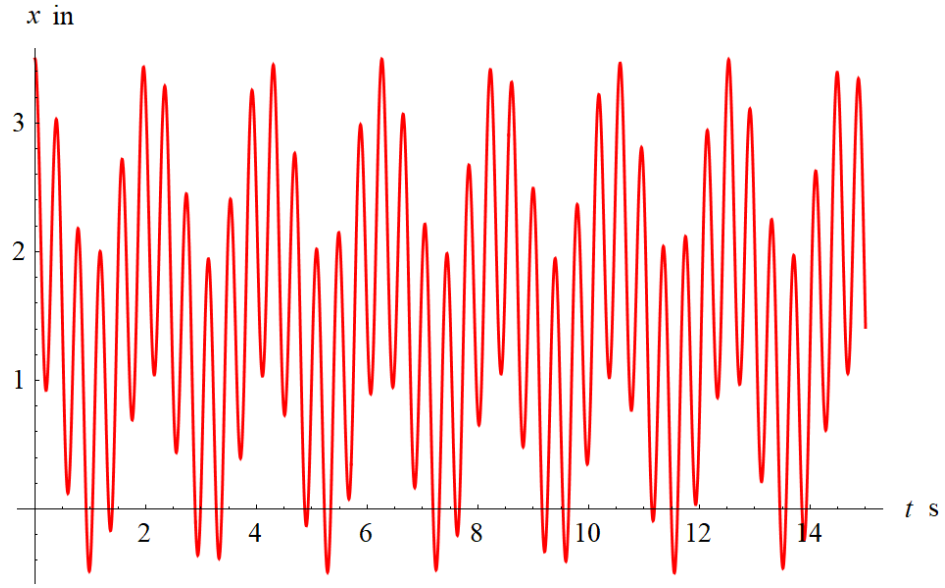
where

$$m = \frac{4 \text{ lb}}{32.2 \ \frac{\text{ft}}{\text{s}^2}}$$
$$k = 32 \ \frac{\text{lb}}{\text{ft}}$$
$$W = 4 \text{ lb},$$

and its solution was found to be

$$x(t) = \left(\frac{7}{24} - \frac{W}{k} - \frac{2}{k - 9m}\right)\cos\sqrt{\frac{k}{m}}t + \frac{W}{k} + \frac{2}{k - 9m}\cos 3t.$$

As $x(t)$ is in feet, multiply the result by 12 to convert it to inches.



Suppose now that the external force is $4\sin\omega t$ rather than $2\cos 3t$. Resonance occurs when ω matches the natural frequency of the system, that is,

$$\omega = \sqrt{\frac{k}{m}} \approx 16.05 \ \frac{\text{rad}}{\text{s}}.$$