Problem 24

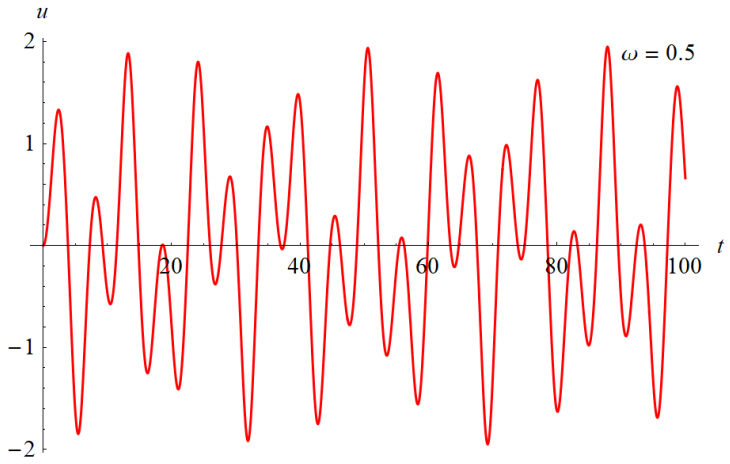
A spring-mass system with a hardening spring (Problem 32 of Section 3.7) is acted on by a periodic external force. In the absence of damping, suppose that the displacement of the mass satisfies the initial value problem

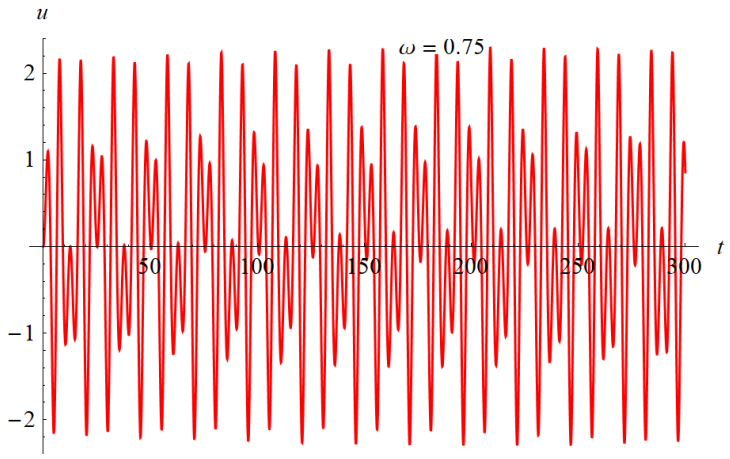
$$u'' + u + \frac{1}{5}u^3 = \cos \omega t, \qquad u(0) = 0, \quad u'(0) = 0.$$

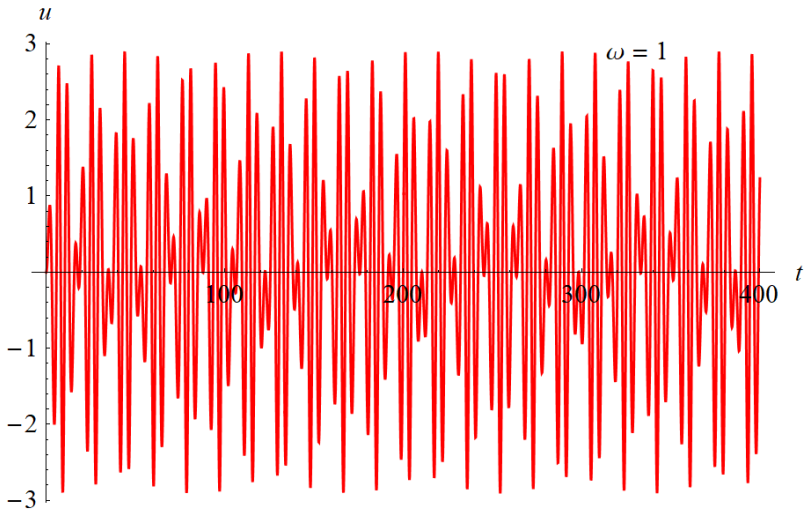
(a) Let $\omega = 1$ and plot a computer-generated solution of the given problem. Does the system exhibit a beat?

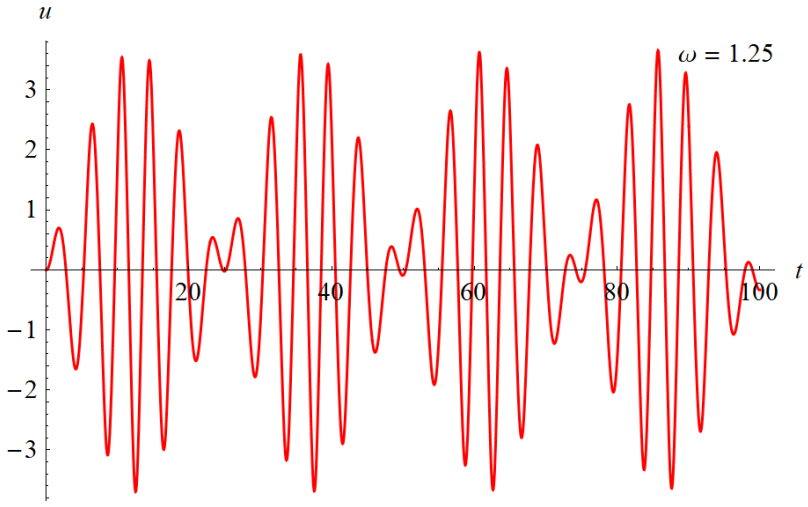
(b) Plot the solution for several values of ω between $1/2$ and 2. Describe how the solution changes as ω increases.

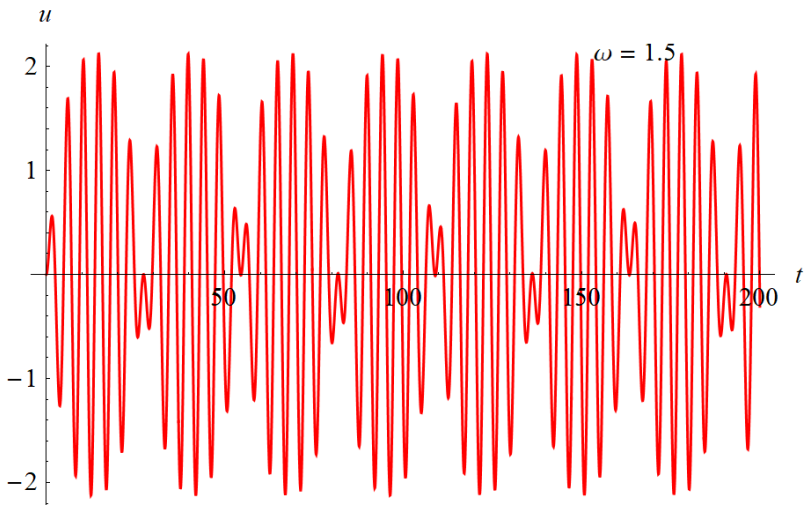
Solution

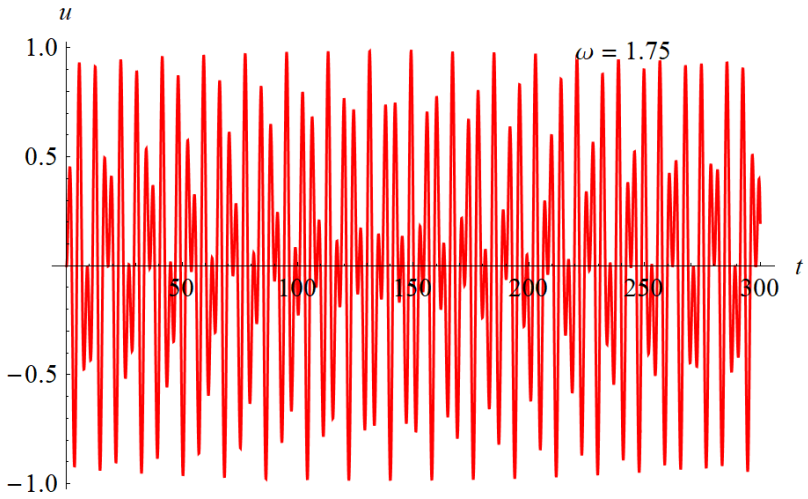


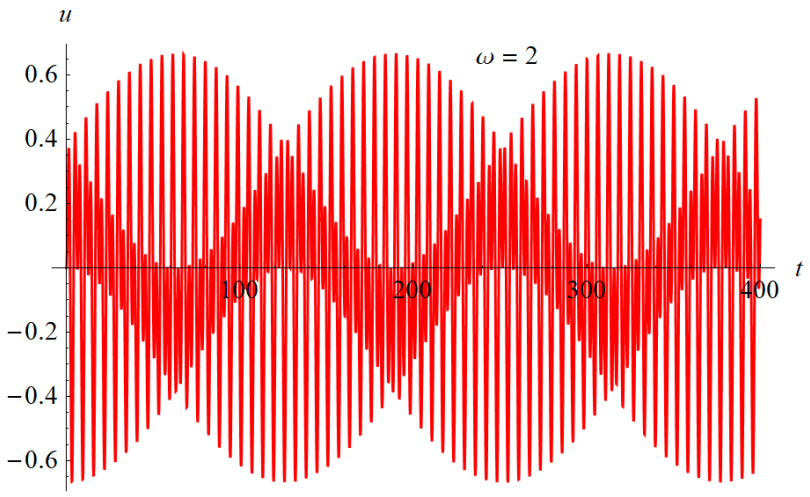



u
ω = 1
t
3
2
1
−1
−2
−3
100
200
300
400


u
ω = 1.25
t
3
2
1
−1
−2
−3
20
40
60
80
100


u
ω = 1.5
2
1
−1
−2
50
100
150
200
t


u
ω = 1.75
1.0
0.5
−0.5
−1.0
50
100
150
200
250
300
t



Beats only seem to occur when $\omega = 1.25$, $\omega = 1.5$, and $\omega = 2$. As ω increases, it's necessary to expand the time scale in order to see the beats.