

Problem 3

In each of Problems 1 through 4, write the given expression as a product of two trigonometric functions of different frequencies.

$$\cos \pi t + \cos 2\pi t$$

Solution

Recall the sum-to-product formula,

$$\cos u + \cos v = 2 \cos \left(\frac{u+v}{2} \right) \cos \left(\frac{u-v}{2} \right).$$

Using this, the given expression becomes

$$\begin{aligned} \cos \pi t + \cos 2\pi t &= 2 \cos \left(\frac{\pi t + 2\pi t}{2} \right) \cos \left(\frac{\pi t - 2\pi t}{2} \right) \\ &= 2 \cos \frac{3\pi t}{2} \cos \frac{\pi t}{2}. \end{aligned}$$