

Problem 9

In each of Problems 7 through 10, determine whether the given functions are linearly dependent or linearly independent. If they are linearly dependent, find a linear relation among them.

$$f_1(t) = 2t - 3, \quad f_2(t) = t^2 + 1, \quad f_3(t) = 2t^2 - t, \quad f_4(t) = t^2 + t + 1$$

Solution

Form the linear combination of f_1 , f_2 , f_3 and f_4 .

$$C_1f_1 + C_2f_2 + C_3f_3 + C_4f_4 = C_1(2t - 3) + C_2(t^2 + 1) + C_3(2t^2 - t) + C_4(t^2 + t + 1)$$

Set it equal to zero.

$$C_1(2t - 3) + C_2(t^2 + 1) + C_3(2t^2 - t) + C_4(t^2 + t + 1) = 0 \tag{1}$$

$$(-3C_1 + C_2 + C_4) + (2C_1 - C_3 + C_4)t + (C_2 + 2C_3 + C_4)t^2 = 0 + 0t + 0t^2$$

Match the coefficients.

$$-3C_1 + C_2 + C_4 = 0$$

$$2C_1 - C_3 + C_4 = 0$$

$$C_2 + 2C_3 + C_4 = 0$$

Solving this system of equations yields $C_2 = 13C_1/2$ and $C_3 = -3C_1/2$ and $C_4 = -7C_1/2$, where C_1 is arbitrary. Since equation (1) can be satisfied by setting $C_1 = 2$ and $C_2 = 13$ and $C_3 = -3$ and $C_4 = -7$, for example, f_1 and f_2 and f_3 and f_4 are linearly dependent.