Problem 12

In each of Problems 11 through 16, verify that the given functions are solutions of the differential equation, and determine their Wronskian.

$$y^{(4)} + y'' = 0; \qquad 1, \quad t, \quad \cos t, \quad \sin t$$

Solution

Check that the first solution satisfies the ODE.

$$(1)^{(4)} + (1)'' \stackrel{?}{=} 0$$

$$(0) + (0) \stackrel{?}{=} 0$$

$$0 = 0$$

Now check that the second solution satisfies the ODE.

$$(t)^{(4)} + (t)'' \stackrel{?}{=} 0$$

$$(0) + (0) \stackrel{?}{=} 0$$

$$0 = 0$$

Now check that the third solution satisfies the ODE.

$$(\cos t)^{(4)} + (\cos t)'' \stackrel{?}{=} 0$$

$$(\cos t) + (-\cos t) \stackrel{?}{=} 0$$

$$0 = 0$$

Now check that the fourth solution satisfies the ODE.

$$(\sin t)^{(4)} + (\sin t)'' \stackrel{?}{=} 0$$

$$(\sin t) + (-\sin t) \stackrel{?}{=} 0$$

$$0 = 0$$

The Wronskian of the four functions is

$$W(1, t, \cos t, \sin t) = \begin{vmatrix} 1 & t & \cos t & \sin t \\ (1)' & (t)' & (\cos t)' & (\sin t)' \\ (1)'' & (t)'' & (\cos t)'' & (\sin t)'' \\ (1)''' & (t)''' & (\cos t)''' & (\sin t)''' \end{vmatrix} = \begin{vmatrix} 1 & t & \cos t & \sin t \\ 0 & 1 & -\sin t & \cos t \\ 0 & 0 & -\cos t & -\sin t \\ 0 & 0 & \sin t & -\cos t \end{vmatrix} = 1[1(\cos^2 t + \sin^2 t)] = 1.$$