

## Problem 15

In each of Problems 11 through 16, verify that the given functions are solutions of the differential equation, and determine their Wronskian.

$$xy''' - y'' = 0; \quad 1, \quad x, \quad x^3$$

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### Solution

Check that the first solution satisfies the ODE.

$$x(1)''' - (1)'' \stackrel{?}{=} 0$$

$$x(0) - (0) \stackrel{?}{=} 0$$

$$0 = 0$$

Now check that the second solution satisfies the ODE.

$$x(x)''' - (x)'' \stackrel{?}{=} 0$$

$$x(0) - (0) \stackrel{?}{=} 0$$

$$0 = 0$$

Now check that the third solution satisfies the ODE.

$$x(x^3)''' - (x^3)'' \stackrel{?}{=} 0$$

$$x(6) - (6x) \stackrel{?}{=} 0$$

$$0 = 0$$

The Wronskian of the three functions is

$$\begin{aligned} W(1, x, x^3) &= \begin{vmatrix} 1 & x & x^3 \\ (1)' & (x)' & (x^3)' \\ (1)'' & (x)'' & (x^3)'' \end{vmatrix} \\ &= \begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & 3x^2 \\ 0 & 0 & 6x \end{vmatrix} \\ &= 6x. \end{aligned}$$