

## Problem 18

Verify that the differential operator defined by

$$L[y] = y^{(n)} + p_1(t)y^{(n-1)} + \cdots + p_n(t)y$$

is a linear differential operator. That is, show that

$$L[c_1y_1 + c_2y_2] = c_1L[y_1] + c_2L[y_2],$$

where  $y_1$  and  $y_2$  are  $n$ -times-differentiable functions and  $c_1$  and  $c_2$  are arbitrary constants. Hence, show that if  $y_1, y_2, \dots, y_n$  are solutions of  $L[y] = 0$ , then the linear combination  $c_1y_1 + \cdots + c_ny_n$  is also a solution of  $L[y] = 0$ .