Problem 5

In each of Problems 1 through 6, determine intervals in which solutions are sure to exist.

$$(x-1)y^{(4)} + (x+1)y'' + (\tan x)y = 0$$

Solution

Divide both sides by $x-1$ so that the coefficient of the highest derivative is 1.

$$y^{(4)} + \frac{x+1}{x-1}y'' + \frac{\sin x}{(x-1)\cos x}y = 0$$

The points of discontinuity are $x = 1$ and

$$\cos x = 0 \quad \rightarrow \quad x = (2n+1)\frac{\pi}{2}, \quad n = 0, \pm 1, \pm 2, \ldots,$$

so depending when the initial conditions are given, the solution to this ODE will be valid either for

$$[2(n-1)+1]\frac{\pi}{2} < x < (2n+1)\frac{\pi}{2}, \quad n = \pm 1, \pm 2, \ldots \quad \text{or} \quad -\frac{\pi}{2} < x < 1 \quad \text{or} \quad 1 < x < \frac{\pi}{2}.$$