

Problem 7

In each of Problems 7 through 10, determine whether the given functions are linearly dependent or linearly independent. If they are linearly dependent, find a linear relation among them.

$$f_1(t) = 2t - 3, \quad f_2(t) = t^2 + 1, \quad f_3(t) = 2t^2 - t$$

Solution

Form the linear combination of f_1 , f_2 , and f_3 .

$$C_1f_1 + C_2f_2 + C_3f_3 = C_1(2t - 3) + C_2(t^2 + 1) + C_3(2t^2 - t)$$

Set it equal to zero.

$$\begin{aligned} C_1(2t - 3) + C_2(t^2 + 1) + C_3(2t^2 - t) &= 0 \\ (-3C_1 + C_2) + (2C_1 - C_3)t + (C_2 + 2C_3)t^2 &= 0 + 0t + 0t^2 \end{aligned} \tag{1}$$

Match the coefficients.

$$\begin{aligned} -3C_1 + C_2 &= 0 \\ 2C_1 - C_3 &= 0 \\ C_2 + 2C_3 &= 0 \end{aligned}$$

Solving this system of equations yields $C_1 = 0$ and $C_2 = 0$ and $C_3 = 0$. Since equation (1) is only satisfied for this trivial selection of C_1 , C_2 , and C_3 , f_1 and f_2 and f_3 are linearly independent.