Problem 2

In each of Problems 1 through 6, express the given complex number in the form $R(\cos\theta + i\sin\theta) = Re^{i\theta}$.

$$-1+\sqrt{3}i$$

Solution

Use Euler's formula to write $e^{i\theta}$ in terms of sine and cosine.

$$\begin{aligned}-1+\sqrt{3}i &= Re^{i\theta}\\ &= R(\cos\theta + i\sin\theta)\\ &= R\cos\theta + iR\sin\theta\end{aligned}$$

Match the coefficients to obtain a system of equations for R and θ.

$$R\cos\theta = -1 \tag{1}$$

$$R\sin\theta = \sqrt{3} \tag{2}$$

To determine R, square both sides of each equation

$$R^2\cos^2\theta = 1$$
$$R^2\sin^2\theta = 3$$

and then add the respective sides.

$$R^2\cos^2\theta + R^2\sin^2\theta = 1+3$$

$$R^2 = 4$$

$$R = 2$$

Divide both sides of equation (2) by the respective sides of equation (1).

$$\tan\theta = -\sqrt{3}$$

$$\theta = \frac{2\pi}{3} + 2n\pi, \quad n = 0, \pm1, \pm2, \ldots$$

Note that adding any multiple of 2π does not change the point's position on the xy-plane. Therefore,

$$-1+\sqrt{3}i = 2e^{i(2\pi/3+2n\pi)}.$$