

### Problem 3

In each of Problems 1 through 6, express the given complex number in the form  $R(\cos \theta + i \sin \theta) = Re^{i\theta}$ .

$$-3$$

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#### Solution

Use Euler's formula to write  $e^{i\theta}$  in terms of sine and cosine.

$$\begin{aligned} -3 &= Re^{i\theta} \\ &= R(\cos \theta + i \sin \theta) \\ &= R \cos \theta + iR \sin \theta \end{aligned}$$

Match the coefficients to obtain a system of equations for  $R$  and  $\theta$ .

$$R \cos \theta = -3 \tag{1}$$

$$R \sin \theta = 0 \tag{2}$$

To determine  $R$ , square both sides of each equation

$$R^2 \cos^2 \theta = 9$$

$$R^2 \sin^2 \theta = 0$$

and then add the respective sides.

$$R^2 \cos^2 \theta + R^2 \sin^2 \theta = 9 + 0$$

$$R^2 = 9$$

$$R = 3$$

Divide both sides of equation (2) by the respective sides of equation (1).

$$\tan \theta = 0$$

$$\theta = \pi + 2n\pi, \quad n = 0, \pm 1, \pm 2, \dots$$

Note that adding any multiple of  $2\pi$  does not change the point's position on the  $xy$ -plane. Therefore,

$$-3 = 3e^{i(\pi+2n\pi)}.$$