Problem 1

In each of Problems 1 through 6, express the given complex number in the form $R(\cos\theta + i\sin\theta) = Re^{i\theta}$.

$$1 + i$$

Solution

Use Euler's formula to write $e^{i\theta}$ in terms of sine and cosine.

$$\begin{aligned} 1 + i &= Re^{i\theta} \\ &= R(\cos\theta + i\sin\theta) \\ &= R\cos\theta + iR\sin\theta \end{aligned}$$

Match the coefficients to obtain a system of equations for R and θ.

$$R\cos\theta = 1 \quad (1)$$

$$R\sin\theta = 1 \quad (2)$$

To determine R, square both sides of each equation

$$R^2\cos^2\theta = 1$$

$$R^2\sin^2\theta = 1$$

and then add the respective sides.

$$R^2\cos^2\theta + R^2\sin^2\theta = 1 + 1$$

$$R^2 = 2$$

$$R = \sqrt{2}$$

Divide both sides of equation (2) by the respective sides of equation (1).

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4} + 2n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$$

Note that adding any multiple of 2π does not change the point's position on the xy-plane. Therefore,

$$1 + i = \sqrt{2}e^{i(\pi/4 + 2n\pi)}.$$