Problem 10

In each of Problems 7 through 10, follow the procedure illustrated in Example 4 to determine the indicated roots of the given complex number.

$$[2(\cos \pi/3 + i \sin \pi/3)]^{1/2}$$

Solution

Write sine and cosine in terms of an exponential function by using Euler's formula.

$$\begin{aligned} [2(\cos \pi/3 + i \sin \pi/3)]^{1/2} &= (2e^{i\pi/3})^{1/2} = [2e^{i(\pi/3+2n\pi)}]^{1/2}, \quad n = 0, \pm 1, \pm 2, \ldots \\ &= \sqrt{2} e^{i(\pi/6+n\pi)} \end{aligned}$$

The two distinct roots are obtained by setting $n = 0$ and $n = 1$. Other values of n lead to redundant roots.

$$\begin{aligned} n = 0: &\quad [2(\cos \pi/3 + i \sin \pi/3)]^{1/2} = \sqrt{2} e^{i\pi/6} = \sqrt{2}\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + i\frac{1}{2} \\ n = 1: &\quad [2(\cos \pi/3 + i \sin \pi/3)]^{1/2} = \sqrt{2} e^{7i\pi/6} = \sqrt{2}\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2} - i\frac{1}{2} \end{aligned}$$