Problem 13

In each of Problems 11 through 28, find the general solution of the given differential equation.

$$2y''' - 4y'' - 2y' + 4y = 0$$

Solution

This is a homogeneous ODE with constant coefficients, so the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2e^{rt} \quad \rightarrow \quad y''' = r^3e^{rt}$$

Substitute these expressions into the ODE.

$$2(r^3e^{rt}) - 4(r^2e^{rt}) - 2(re^{rt}) + 4(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$2r^3 - 4r^2 - 2r + 4 = 0$$

$$2(r-2)(r-1)(r+1) = 0$$

$$r = \{-1, 1, 2\}$$

Three solutions to the ODE are then $y = e^{-t}$ and $y = e^t$ and $y = e^{2t}$. By the principle of superposition, the general solution is a linear combination of these three. Therefore,

$$y(t) = C_1e^{-t} + C_2e^t + C_3e^{2t}.$$