# Problem 39

Consider the spring-mass system, shown in Figure 4.2.4, consisting of two unit masses suspended from springs with spring constants 3 and 2, respectively. Assume that there is no damping in the system.

(a) Show that the displacements $u_1$ and $u_2$ of the masses from their respective equilibrium positions satisfy the equations

$$u_1'' + 5u_1 = 2u_2, \qquad u_2'' + 2u_2 = 2u_1. \tag{i}$$

(b) Solve the first of Eqs. (i) for $u_2$ and substitute into the second equation, thereby obtaining the following fourth order equation for $u_1$:

$$u_1^{(4)} + 7u_1'' + 6u_1 = 0. \tag{ii}$$

Find the general solution of Eq. (ii).

(c) Suppose that the initial conditions are

$$u_1(0) = 1, \qquad u_1'(0) = 0, \qquad u_2(0) = 2, \qquad u_2'(0) = 0. \tag{iii}$$

Use the first of Eqs. (i) and the initial conditions (iii) to obtain values for $u_1''(0)$ and $u_1'''(0)$. Then show that the solution of Eq. (ii) that satisfies the four initial conditions on $u_1$ is $u_1(t) = \cos t$. Show that the corresponding solution $u_2$ is $u_2(t) = 2\cos t$.

(d) Now suppose that the initial conditions are

$$u_1(0) = -2, \qquad u_1'(0) = 0, \qquad u_2(0) = 1, \qquad u_2'(0) = 0. \tag{iv}$$

Proceed as in part (c) to show that the corresponding solutions are $u_1(t) = -2\cos\sqrt{6}t$ and $u_2(t) = \cos\sqrt{6}t$.

(e) Observe that the solutions obtained in parts (c) and (d) describe two distinct modes of vibration. In the first, the frequency of the motion is 1, and the two masses move in phase, both moving up or down together; the second mass moves twice as far as the first. The second motion has frequency $\sqrt{6}$, and the masses move out of phase with each other, one moving down while the other is moving up, and vice versa. In this mode the first mass moves twice as far as the second. For other initial conditions, not proportional to either of Eqs. (iii) or (iv), the motion of the masses is a combination of these two modes.

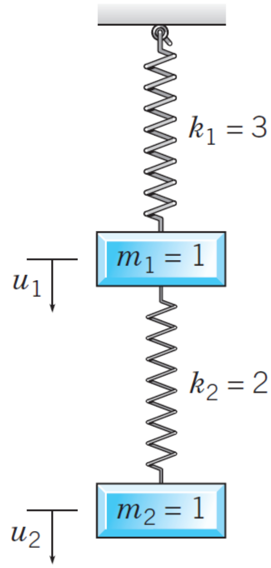


**FIGURE 4.2.4** A two-spring, two-mass system.