

Problem 4

In each of Problems 1 through 6, express the given complex number in the form $R(\cos \theta + i \sin \theta) = Re^{i\theta}$.

$$-i$$

Solution

Use Euler's formula to write $e^{i\theta}$ in terms of sine and cosine.

$$\begin{aligned} -i &= Re^{i\theta} \\ &= R(\cos \theta + i \sin \theta) \\ &= R \cos \theta + iR \sin \theta \end{aligned}$$

Match the coefficients to obtain a system of equations for R and θ .

$$R \cos \theta = 0 \tag{1}$$

$$R \sin \theta = -1 \tag{2}$$

To determine R , square both sides of each equation

$$R^2 \cos^2 \theta = 0$$

$$R^2 \sin^2 \theta = 1$$

and then add the respective sides.

$$R^2 \cos^2 \theta + R^2 \sin^2 \theta = 0 + 1$$

$$R^2 = 1$$

$$R = 1$$

Divide both sides of equation (2) by the respective sides of equation (1).

$$\tan \theta = \infty$$

$$\theta = \frac{3\pi}{2} + 2n\pi, \quad n = 0, \pm 1, \pm 2, \dots$$

Note that adding any multiple of 2π does not change the point's position on the xy -plane. Therefore,

$$-i = e^{i(3\pi/2+2n\pi)}.$$