Problem 5

In each of Problems 1 through 6, express the given complex number in the form $R(\cos\theta + i\sin\theta) = Re^{i\theta}$.

$$\sqrt{3} - i$$

Solution

Use Euler's formula to write $e^{i\theta}$ in terms of sine and cosine.

$$\begin{aligned}\sqrt{3} - i &= Re^{i\theta} \\ &= R(\cos\theta + i\sin\theta) \\ &= R\cos\theta + iR\sin\theta\end{aligned}$$

Match the coefficients to obtain a system of equations for R and θ.

$$R\cos\theta = \sqrt{3} \tag{1}$$

$$R\sin\theta = -1 \tag{2}$$

To determine R, square both sides of each equation

$$\begin{aligned}R^2\cos^2\theta &= 3 \\ R^2\sin^2\theta &= 1\end{aligned}$$

and then add the respective sides.

$$R^2\cos^2\theta + R^2\sin^2\theta = 3 + 1$$

$$R^2 = 4$$

$$R = 2$$

Divide both sides of equation (2) by the respective sides of equation (1).

$$\tan\theta = -\frac{1}{\sqrt{3}}$$

$$\theta = -\frac{\pi}{6} + 2n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$$

Note that adding any multiple of 2π does not change the point's position on the xy-plane. Therefore,

$$\sqrt{3} - i = 2e^{i(-\frac{\pi}{6} + 2n\pi)}.$$