

## Problem 7

In each of Problems 7 through 10, follow the procedure illustrated in Example 4 to determine the indicated roots of the given complex number.

$$1^{1/3}$$

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### Solution

Write 1 in the form of  $Re^{i\theta}$ .

$$\begin{aligned} 1^{1/3} &= (1e^{2in\pi})^{1/3}, \quad n = 0, \pm 1, \pm 2, \dots \\ &= 1^{1/3} e^{2in\pi/3} \\ &= e^{2in\pi/3} \end{aligned}$$

The three distinct roots are obtained by setting  $n = 0$ ,  $n = 1$ , and  $n = 2$ . Other values of  $n$  lead to redundant roots.

$$n = 0 : \quad 1^{1/3} = e^0 = 1$$

$$n = 1 : \quad 1^{1/3} = e^{2i\pi/3} = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$n = 2 : \quad 1^{1/3} = e^{4i\pi/3} = \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$