Problem 9

In each of Problems 7 through 10, follow the procedure illustrated in Example 4 to determine the indicated roots of the given complex number.

$$1^{1/4}$$

Solution

Write 1 in the form of $Re^{i\theta}$.

$$\begin{aligned} 1^{1/4} &= (1e^{2in\pi})^{1/4}, \quad n = 0, \pm 1, \pm 2, \dots \\ &= 1^{1/4} e^{2in\pi/4} \\ &= e^{in\pi/2} \end{aligned}$$

The four distinct roots are obtained by setting $n = 0$, $n = 1$, $n = 2$, and $n = 3$. Other values of n lead to redundant roots.

$$\begin{aligned} n = 0: &\quad 1^{1/4} = e^0 = 1 \\ n = 1: &\quad 1^{1/4} = e^{i\pi/2} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i \\ n = 2: &\quad 1^{1/4} = e^{i\pi} = \cos\pi + i\sin\pi = -1 \\ n = 3: &\quad 1^{1/4} = e^{3i\pi/2} = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i \end{aligned}$$