Problem 19

Consider the nonhomogeneous nth order linear differential equation

$$a_0y^{(n)} + a_1y^{(n-1)} + \cdots + a_ny = g(t), \tag{i}$$

where $a_0, \ldots, a_n$ are constants. Verify that if $g(t)$ is of the form

$$e^{\alpha t}(b_0t^m + \cdots + b_m),$$

then the substitution $y = e^{\alpha t}u(t)$ reduces Eq. (i) to the form

$$k_0u^{(n)} + k_1u^{(n-1)} + \cdots + k_nu = b_0t^m + \cdots + b_m, \tag{ii}$$

where $k_0, \ldots, k_n$ are constants. Determine k_0 and k_n in terms of the a's and α. Thus the problem of determining a particular solution of the original equation is reduced to the simpler problem of determining a particular solution of an equation with constant coefficients and a polynomial for the nonhomogeneous term.