Problem 1

In each of Problems 1 through 8, determine the radius of convergence of the given power series.

$$\sum_{n=0}^{\infty}(x-3)^n$$

Solution

Apply the ratio test to determine the condition in which the provided series converges.

$$\begin{aligned}
\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{(x-3)^n}\right| \\
&= \lim_{n\to\infty}|x-3| \\
&= |x-3|
\end{aligned}$$

According to this test, the series is

$$\begin{cases} \text{convergent} & \text{if } |x-3|<1 \\ \text{unknown} & \text{if } |x-3|=1 \\ \text{divergent} & \text{if } |x-3|>1 \end{cases}.$$

From the condition of convergence, which can also be written as $-1 < x-3 < 1$, or $2 < x < 4$, we see that the center of convergence is at $x = 3$ and the radius of convergence is 1.