Problem 4

In each of Problems 1 through 8, determine the radius of convergence of the given power series.

$$\sum_{n=0}^{\infty} 2^n x^n$$

Solution

Apply the ratio test to determine the condition in which the provided series converges.

$$\begin{aligned}\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty}\left|\frac{2^{n+1}x^{n+1}}{2^n x^n}\right| \\ &= \lim_{n\to\infty}|2x| \\ &= \lim_{n\to\infty} 2|x| \\ &= 2|x|\end{aligned}$$

According to this test, the series is

$$\begin{cases}\text{convergent} & \text{if } 2|x| < 1 \\ \text{unknown} & \text{if } 2|x| = 1 \\ \text{divergent} & \text{if } 2|x| > 1\end{cases}.$$

From the condition of convergence, which can also be written as $|x| < 1/2$, or $-1/2 < x < 1/2$, we see that the center of convergence is at $x = 0$ and the radius of convergence is $1/2$.