

Problem 23

In each of Problems 21 through 27, rewrite the given expression as a sum whose generic term involves x^n .

$$x \sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{k=0}^{\infty} a_k x^k$$

Solution

Bring x inside the summand.

$$\sum_{n=1}^{\infty} n a_n x^n + \sum_{k=0}^{\infty} a_k x^k$$

Substitute $k = n$ in the second sum.

$$\sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} a_n x^n$$

Write out the first term of the second sum so that it starts at $n = 1$ like the first one.

$$\sum_{n=1}^{\infty} n a_n x^n + a_0 x^0 + \sum_{n=1}^{\infty} a_n x^n$$

Combine the two sums.

$$a_0 + \sum_{n=1}^{\infty} (n a_n x^n + a_n x^n)$$

Factor $a_n x^n$.

$$a_0 + \sum_{n=1}^{\infty} (n + 1) a_n x^n$$

Start the sum at $n = 0$ to include a_0 .

$$\sum_{n=0}^{\infty} (n + 1) a_n x^n$$