Problem 24

In each of Problems 21 through 27, rewrite the given expression as a sum whose generic term involves x^n.

$$(1-x^2)\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$$

Solution

Expand this term.

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - x^2\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$$

Bring x^2 into the summand.

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1)a_n x^{n}$$

Substitute $k = n-2$ in the first sum and substitute $k = n$ in the second sum.

$$\sum_{k+2=2}^{\infty} (k+2)(k+1)a_{k+2} x^{k} - \sum_{k=2}^{\infty} k(k-1)a_k x^{k}$$

Solve for k.

$$\sum_{k=0}^{\infty} (k+2)(k+1)a_{k+2} x^{k} - \sum_{k=2}^{\infty} k(k-1)a_k x^{k}$$

Because of the factors, k and $k-1$, the second sum can be started at $k=0$.

$$\sum_{k=0}^{\infty} (k+2)(k+1)a_{k+2} x^{k} - \sum_{k=0}^{\infty} k(k-1)a_k x^{k}$$

Combine the two sums.

$$\sum_{k=0}^{\infty} [(k+2)(k+1)a_{k+2} x^{k} - k(k-1)a_k x^{k}]$$

Factor x^k.

$$\sum_{k=0}^{\infty} [(k+2)(k+1)a_{k+2} - k(k-1)a_k]x^{k}$$

As k is only a dummy index, it can be replaced with n.

$$\sum_{n=0}^{\infty} [(n+2)(n+1)a_{n+2} - n(n-1)a_n]x^{n}$$